On the two-fluid model of the Kondo lattice

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The normal state of conduction electrons in metals at low temperatures has been described in terms of the standard theory of a Fermi liquid introduced soon after the advent of quantum mechanics and completed by Landau and others by the middle of the 20th century (1). Fermi liquid theory describes the nature of a quantum liquid of interacting itinerant fermions below a characteristic temperature T_{FL}, which can be far below the bare Fermi temperature T_F. The state above T_{FL}, but below T_F, is itself a correlated quantum liquid that can extend down to very low temperatures close to a quantum critical point (Fig. 1*A*) (2).

Yang and Pines (3) and Shirer et al. (4) present a phenomenological description of such a correlated quantum liquid known as the Kondo liquid (5). In the simplest case of the Anderson lattice model, a Kondo liquid arises in a system of electrons in a conduction band hybridized with a half-filled and narrow f-band. The effect of the Coulomb interaction is to suppress double occupancy of the f-orbital on each atomic site, leading to Mott localization in the absence of hybridization. The effect of hybridization is to dope the f-band with holes in a way reminiscent of hole doping of a narrow d-band in the copper oxide superconductors via the effects of chemical substitution or pressure (e.g., 5, 6).

The Yang and Pines model (3) of the Kondo liquid has a particularly simple form at the quantum critical point separating the magnetic and nonmagnetic state where T_{FL} vanishes (Fig. 1*A*). In the low temperature limit, the Kondo liquid state is defined by an entropy S and magnetic susceptibility χ of the following form:

$$S \sim T\chi \sim T\, ln(\theta/T) \qquad [1]$$

where θ is a scale that is defined here later. This differs from the Fermi liquid form expected for $T < T_{FL}$ by the logarithmic correction factor $ln(\theta\,/\,T)$, which is characteristic of a marginal Fermi liquid state (7–11). If S/T is considered a measure of the mass of fermionic quasiparticle excitations, the Kondo liquid can be viewed as a state with singular mass renormalization in the limit $T \to 0$ K.

Yang and Pines (3) go beyond this marginal Fermi liquid limit by introducing a two-fluid model to extend the description of the Kondo liquid away from the quantum critical point and up to a temperature scale T^* above which hybridization between the conduction and f-electron states becomes ineffective. In practice, T^* is defined as the temperature at which the entropy reaches $R\, ln\, 2$ per mole of f states if $k_B T^*$ is less than the relevant crystal-field energy splitting.

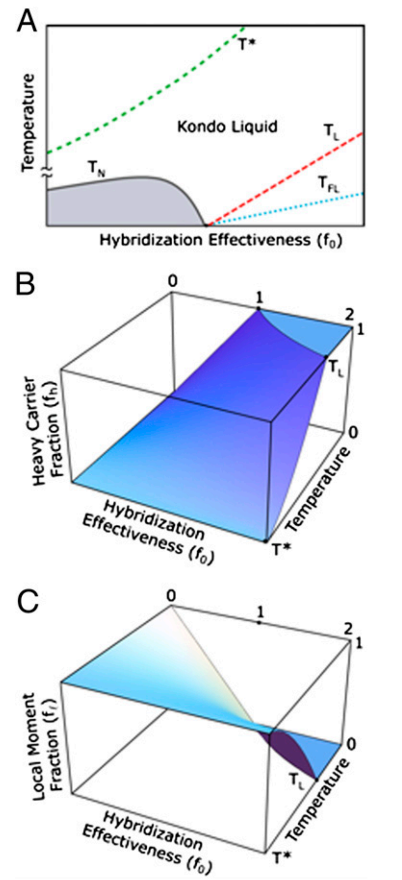

Fig. 1. Two-fluid model of the Kondo lattice. (*A*) Schematic temperature-vs.-hybridization effectiveness phase diagram about a quantum critical point (midpoint of lower horizontal axis). f_0 can be tuned in practice via hydrostatic pressure or chemical substitution. T_N is the Néel temperature and T^* is the hybridization crossover temperature. (*B* and *C*) the heavy carrier fraction $f_h(T)$ and local moment fraction $f_l(T)$, respectively, as functions of temperature and hybridization effectiveness in the two-fluid model. The quantum critical point corresponds to a T of 0 and an f_0 of 1. The heavy carrier fraction saturates and the local moment fraction vanishes below T_L for $f_0 > 1$. The Kondo liquid tends to be unstable to superconductivity or other subtle forms of quantum order. The transitions or crossovers into such states as well as the relocalization temperature just above T_N (and T_N itself in *B* and *C*) are not indicated.

In the two-fluid model, one fluid represents "heavy fermion carriers" of fraction $f_h(T)$ and the other fluid represents "local moments" of fraction

$$f_l(T) = 1 - f_h(T) \qquad [2]$$

where $f_h(T)$ is taken to be of the following form:

$$f_h = f_0(1 - T/T^*)^{3/2} \qquad [3]$$

The coefficient f_0 is called the hybridization effectiveness and plays a crucial role in the description of the Kondo liquid state away from the quantum critical point (Fig. 1). For $f_0 > 1$, the local moment fraction vanishes and the heavy carrier fraction reaches unity at a finite temperature T_L. Below a still lower temperature T_{FL} (Fig. 1*A*), the system condenses into a Fermi liquid state characterized by fermion quasiparticles of strongly enhanced but nonsingular masses excited about a Fermi surface importantly enclosing hybridized *f*-electron and conduction electron states (e.g., 12). For $f_0 < 1$, on the contrary, the local moment fraction fails to vanish at any temperature. Local moments then coexist with heavy carriers that mediate a magnetic interaction analogous to the Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction (5). This interaction leads to antiferromagnetic order at finite temperatures (below the Nèel temperature, T_N, in Fig. 1*A*). Close to and above T_N, the local moment fraction deviates from the above model as a result of "relocalization," i.e., a precursor to magnetic order.

Thus, the two-fluid model describes the Kondo liquid essentially in terms of two universal exponents, namely a three-halves exponent defining the temperature dependence of the heavy carrier fraction $f_h(T)$, and a marginally sublinear exponent defining the temperature dependence of the heavy carrier entropy and susceptibility times temperature. The corresponding contribution of local moments involves no new universal functions and is given by $[1 - f_h(T)]$ times $R\, ln\, 2$ for the molar entropy and $[1 - f_h(T)]$ times the Curie constant for the susceptibility times temperature.

This remarkably simple two-fluid description has been used to correlate a large

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body of data in 4f- and 5f-electron materials on the border of antiferromagnetism at low temperatures (refs. 3 and 4 and references therein). Besides the entropy and magnetic susceptibility, the model has also been used, for example, to describe the temperature dependence of the NMR Knight shift and, with the inclusion of the RKKY interaction, the temperature dependence of the NMR relaxation rate. Interestingly, the magnetic susceptibility and Knight shift are not expected to have equivalent temperature dependences because the heavy carrier and local moments couple in different ways to the nuclear moments. This difference provides a way of thinking about the experimentally observed and puzzling deviations reported between the behavior of the Knight shift and the bulk susceptibility in a number of 4f and 5f systems.

The phenomenological two-fluid model is also consistent with some microscopic descriptions. In particular, the marginally sublinear form of the heavy carrier entropy at low temperatures has arisen in models including effects of singular fermion interactions associated with spin, gauge, or hybridization field fluctuations (7–11). These have led to local and nonlocal marginal Fermi liquid descriptions, the former being the more relevant to the low-temperature limit of the two-fluid model of Yang and Pines (3). Also, the extended form of the heavy carrier density of states defined via the ratio of the entropy to

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the temperature in the two-fluid model, i.e., $ln(\theta / T)(1 - T / T^*)^{3/2}$, with θ equal to eT^*, has been found to follow closely that predicted by dynamical mean field theory (13, 14).

The quantities $f_h(T)$ and $f_l(T)$ are, at first sight, analogous to the normalized superfluid and normal densities, respectively, in the two-fluid model of liquid He II (15). In contrast to the latter, however, there is no change in symmetry and no well-defined transition at T^*. Apart from T_N, the temperature scales in Fig. 1*A* are all crossover temperatures. Also, the Kondo liquid state tends to be unstable to the formation of superconductivity and possibly other exotic forms of long-range order or types of quantum liquids in the low temperature limit. The effective RKKY interaction introduced in the two-fluid model would seem to operate only in the presence of a finite local moment fraction and hence only for $f_0 < 1$ in the zero temperature limit. Additional phenomenological effective interactions (e.g., 5, 16) would therefore need to be introduced to describe instabilities for $f_0 \geq 1$ and hence to provide a more complete picture of the Kondo liquid state.

The two-fluid model of Yang and Pines (3) and Shirer et al. (4) provides a simple operational way of thinking about a complex cooperative state of matter, the Kondo liquid. As in the case of the two-fluid model of He II (15), a microscopic theory is needed to clarify unambiguously the meaning of the two fluids and further to understand the origins of the universal behavior of the Kondo liquid in the wider context of quantum phase transitions in d- and f-electron systems.

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