Quantum critical scaling and superconductivity in heavy electron materials

Yi-feng Yang,1,2,* David Pines,3,4 and N. J. Curro3

1Beijing National Laboratory for Condensed Matter Physics and Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China
2Collaborative Innovation Center of Quantum Matter, Beijing 100190, China
3Department of Physics, University of California, Davis, California 95616, USA
4Santa Fe Institute, Santa Fe, New Mexico 87501, USA

(Received 2 October 2014; revised manuscript received 2 November 2015; published 17 November 2015)

We use the two fluid model to determine the conditions under which the nuclear spin-lattice lattice relaxation rate \( T_1 \) of candidate heavy quantum critical superconductors can exhibit scaling behavior and find that it can occur if and only if their “hidden” quantum critical spin fluctuations give rise to a temperature-independent intrinsic heavy electron spin-lattice relaxation rate. The resulting scaling of \( T_1 \) with the strength of the heavy electron component and the coherence temperature \( T^* \) provides a simple test for their presence at pressures at which the superconducting transition temperature \( T_c \) is maximum and is proportional to \( T^* \). These findings support the previously noted partial scaling of the spin-lattice relaxation rate with \( T_c \) in a number of important heavy electron materials and provide additional evidence that in these materials their optimal superconductivity originates in the quantum critical spin fluctuations associated with a nearby phase transition from partially localized to fully itinerant quasiparticles.

DOI: 10.1103/PhysRevB.92.195131

PACS number(s): 71.27.+a, 74.70.Tx, 76.60.–k

A tantalizing hint that the spin fluctuations seen in the nuclear spin relaxation rate for a number of unconventional superconductors might be the magnetic glue responsible for their superconductivity appears in a scaling relation between that rate and the optimal superconducting transition temperature \( T_c \) that was first noted by Curro et al. [1]. In this paper we focus on understanding this scaling relation for one important member of this family, the heavy electron materials, for which some experimental results are given in Fig. 1 [1,2]. As may be seen in Fig. 2, finding such a relation appears at first sight highly problematic because the scaling covers a range of temperatures \( (T_c < T < T^*) \) in the normal state in which both hybridized localized spins and the itinerant heavy electron Kondo liquid contribute to the spin-lattice relaxation rate. However, we find that rigorous Curro \( T_c \) scaling can become possible if three conditions are met: (1) the maximum in \( T_c \) occurs at the pressure \( p_L \), at which the line marking the boundary between partially localized and fully itinerant behavior for heavy electron quasiparticles \( T_L \) intersects with \( T_c \), so that \( T_c^{\max} = T_L(p_L) \); (2) at \( p_L \) the total spin-lattice relaxation rate scales with the coherence temperature \( T^*(p_L) \); and (3) at \( p_L \) the materials possess identical values of the product \( (3T_c/2T^*) (T^{KL}_L/I^{SL}_L) \), where \( T^{KL}_L \) and \( T^{SL}_L \) are the intrinsic temperature independent spin-lattice relaxation rates of the Kondo and spin liquids, respectively.

If the first condition is met, \( T_c \) will scale with the coherence temperature \( T^*(p_L) \) [3]. The second scaling relation requires that the intrinsic spin-lattice relaxation rate of the heavy electron Kondo liquid be independent of temperature over much of the relevant temperature region, which will be the case if, and only if, it is caused by magnetic quantum critical spin fluctuations [4]. We present a simple way to test whether the second condition is met and find that it is satisfied for a number of heavy electron superconductors. The third condition tells us that Curro scaling cannot be universal, since it involves two material-specific parameters, \( 3T_c/2T^* \) and \( T^{KL}_L/I^{SL}_L \), but, as may be seen in Fig. 1, these ratios are sufficiently similar for a number of materials that Curro scaling is approximately valid.

The phenomenological two-fluid model, which explains so many other aspects of heavy electron behavior [4–12], is also key to understanding the remarkable scaling of their dynamic behavior. In it, below \( T^* \), the collective hybridization of the Kondo lattice of \( f \)-electron local moments with the background conduction electrons gives rise to an itinerant heavy electron Kondo liquid (KL) of strength (or volume fraction) \( f \) that coexists with the hybridized local moment spin liquid of strength \( 1 − f \) with [5]

\[
f(T,p) = f_0(p) \left(1 - \frac{T}{T^*}\right)^{3/2}.
\]

The parameter \( f_0(p) \) provides a quantitative measure of hybridization effectiveness; as may be seen in Fig. 2, it is unity at the delocalization quantum critical point (QCP) that marks a zero temperature transition from partially localized to fully itinerant behavior, and it typically increases with increasing pressure for Ce-based compounds. For \( f_0 > 1 \), the model enables one to determine the pressure and temperature dependence of the delocalization line [5] \( T_L \), shown in Fig. 2, that begins at the delocalization QCP and marks, at finite temperatures, the end of the collective hybridization process at \( f(T_L,p) = 1 \). It follows from Eq. (1) that the delocalization temperature takes the form

\[
T_L(p) = T^*(p)[1 - f_0(p)^{-2/3}].
\]

Since partial localization (\( f < 1 \)) competes with superconductivity, the two-fluid model predicts that the maximum in the superconducting transition temperature \( T^{\max} \) will be found at the pressure \( p_L \) at which the delocalization line \( T_L \) intersects \( T_c \); experiments on CeCoIn_5 and CeRhIn_5 [13,14] show that this is the case for these materials for

*yifeng@iphy.ac.cn
which $p_L = 1.4$ and 2.4 GPa, respectively. For pressures less than $p_L$, partial quasiparticle localization reduces the number of heavy electrons able to become superconducting and so suppresses $T_c$, while for pressures greater than $p_L$, it is physically appealing to assume that the attractive interaction between heavy electron quasiparticles becomes increasingly less effective [15].

Note that the quantum critical pressure $p_{QC} \neq p_L$, as shown in Fig. 2. However, for candidate quantum critical superconductors, $f_0(p_L)$ is not far from its value at the quantum critical point $f_0(p_{QC}) = 1$, so we can obtain a simple relation between $T_c^\text{max}$ and $T^*(p_L)$.

$$T_c^\text{max} = f_0(p_L) - 2/3 \approx 2/3[f_0(p_L) - 1].$$  
(3)

Since $f_0(p_L)$ is a material sensitive parameter, we see that the ratio of $T_c$ to $T^*$ will in general vary from one material to another and that $T_c/T^*$ scaling will occur only for materials with comparable values of $f_0(p_L)$. This is roughly the case for the materials shown in Table I, where the value of $f_0$ is estimated from other experiments for CeCoIn$_5$, CeRhIn$_5$, and URu$_2$Si$_3$ and assumed to be near unity at the optimal pressure $p_L$ for PuCoGa$_5$.

We turn next to the two-fluid expression for the spin-lattice relaxation rate [4],

$$\frac{1}{T_1} = \frac{1 - f(T)}{T_1^{SL}} + \frac{f(T)}{T_1^{KL}},$$  
(4)

where $T_1^{SL}$ and $T_1^{KL}$ are the intrinsic spin-lattice relaxation times of the hybridized local moment spin liquid and the itinerant Kondo liquid. Equation (4) tells us that $T/T^*$ scaling will be found if and only if both intrinsic spin-lattice relaxation rates are independent of temperature, with all the measured temperature dependence of $1/T_1$ originating in the strength of the heavy electron component $f(T)$. For the spin liquid, experiment suggests that $1/T_1^{SL}$ becomes almost temperature independent as one approaches $T^*$ for materials that are at or near the quantum critical or localization pressure (cf. the inset in Fig. 1), and it is reasonable to assume that this behavior continues below $T^*$. Importantly, the intrinsic Kondo liquid spin-lattice rate will be nearly independent of temperature over a wide range of temperatures provided the spin fluctuations responsible for its spin-lattice relaxation rate, $1/T_1^{KL}$, exhibit the magnetic quantum critical behavior [4,19] that has been shown to give rise to $T_1^{KL}/T \sim (T + T_0)$ with $T_0$ being small and going to zero at the QCP.

For any material for which $T^*$ is known, it is straightforward to test the extent to which these conditions are met for the intrinsic spin-lattice relaxation rates: One has only to plot $1/T_1$ vs the strength of the heavy electron component $(1 - T/T^*)^{3/2}$ for temperatures below $T^*$. Upon rewriting Eq. (4) as

$$\frac{1}{T_1} = \frac{1}{T_1^{SL}} + \left(\frac{1}{T_1^{KL}} - \frac{1}{T_1^{SL}}\right)f_0(p_L)\left(1 - \frac{T}{T^*}\right)^{3/2},$$  
(5)

we see that to the extent that one finds linear behavior, one can determine directly both $1/T_1^{SL}$ and the product $f_0(p_L)(1/T_1^{KL} - 1/T_1^{SL})$ from such a scaling plot. Our results for a number of heavy electron materials are given in Fig. 3, where the expected linear scaling is found between $T^*$ and a cutoff temperature $T_c$ for each material shown there, while the corresponding results for the intrinsic spin lattice relaxation rates are given in Table I.
We further note that to the extent that the ratio $T_{KL}/T_{SL}$

is similar for different superconductors, one can obtain two simple nonuniversal scaling formulas for the spin-lattice relaxation rate. The first, applicable at temperatures from $T_*$ up to $T_*/2$, concerns the scaling behavior with $T_*$ depicted in Fig. 1. Assuming two-fluid scaling behavior persists down to $T_*$, the strength of the heavy electron component is given by $f(T) = f_0(p_L)(1 - 3T/2T_*) \approx 1 - 3(T - T_c)/2T_*$ at $p_L$, and one easily finds

$$T_1(T) = f(T) + [1 - f(T)]T_{KL}/T_{SL}$$

$$\approx 1 + g^*(T/T_c^* - 1),$$

where $g^*$ is a nonuniversal parameter and equal to $(3T_*/2T^*)(T_{KL}/T_{SL} - 1)$, which is calculated and listed in Table I for each material. A suggested scaling line with $g^* = 0.27$ is plotted in Fig. 1 for comparison with experiment. We see that the overall agreement looks good despite the lack of universality.

A second scaling expression emerges if one considers the spin-lattice relaxation rate normalized at a fixed temperature, say $0.2T^*$:

$$T_1(0.2T^*)/T_1(T) \approx t_0 + 1.4(1 - t_0)(1 - T/T_c^*)^{3/2},$$

where $t_0 = T_1(0.2T^*)/T_1(T^*)$. As shown in Table I, $t_0 \approx 2.7$ for PuCoGa$_5$ and CeCoIn$_5$ and is somewhat lower for the other member of the Ce115 family, CeRhIn$_5$. The solid lines in Fig. 4(a) shows that $t_0 = 2.7$ provides a surprisingly good fit to the data for the materials shown there, despite the fact that the hyfperfine coupling constants and spin liquid contributions may differ from material to material, and not all of these are at the pressure $p_L$ at which $T_*$ is maximum.

We now look into the details of the scaling behavior for individual materials. In Fig. 3(a), one sees that the $T^*$ scaling continues down to the superconducting transition in CeCoIn$_5$ ($p = 1.2$ GPa) and PuCoGa$_5$. This proves that for CeCoIn$_5$ at $p_L$ the spin fluctuations responsible for the intrinsic Kondo liquid $T_1$ exhibit quantum critical magnetic behavior and suggests a possibly similar situation in PuCoGa$_5$. We note that recent elastic moduli measurements have observed an anomalous softening of the bulk modulus in PuCoGa$_5$ and suggested that valence fluctuations may play a critical role in its superconductivity [20]. Combining their results and the NMR measurement [1] suggests that both valence and spin fluctuations may be involved to give rise to the particularly high $T_*$ in this interesting compound.

For CeRhIn$_5$ at $p = 2.7$ GPa, a pressure that is larger than its experimental value of $p_L = 2.4$ GPa, we see that scaling behavior is found down to a temperature $T_2 \approx T_1$, its delocalization temperature, that is calculated to be 5.7 K using $T^* = 42$ K and $f_0(2.7$ GPa) = 1.24 estimated from other experiments [15]; the loss of scaling below $T_2$ must be attributed to the intrinsic Kondo liquid. As may be seen

![Graph](195131-3)
from a fully formed itinerant heavy electron Kondo liquid and that its physical origin is likely an attractive quasiparticle interaction induced by their coupling to quantum critical spin fluctuations, an interaction that, in this case, leads to the hidden order state, rather than superconductivity at much lower temperatures.

The behavior of the spin-lattice relaxation rate of both CeCu$_2$Si$_2$ and YbRh$_2$Si$_2$ at ambient pressure is anomalous. Although a number of experiments have suggested a $T^* \sim 70$ K for CeCu$_2$Si$_2$, we see in Fig. 3(b) that its spin-lattice relaxation rate $1/T_1$ continues to be temperature independent down to $\sim 8$ K [17], below which spin density wave fluctuations are observed in neutron scattering experiments [21]. We encounter a similar anomaly in YbRh$_2$Si$_2$ for which $T^* \sim 50$ K [18,22].

Assuming the two-fluid model is applicable, this either requires a nonaccidental cancellation ($T^L_K \approx T^K_L$) of the independent spin liquid and Kondo liquid contributions to $1/T_1$ over a substantial temperature range ($T > T_0$) or a quite different physical picture for spin-lattice relaxation in these two materials. At this stage, we cannot make explicit predictions on the nature of the QCP in each material.

In summary, we have shown that an analysis based on the two-fluid model leads to a number of nonuniversal quantum critical scaling relations in the spin-lattice relaxation rate of heavy electron materials at the pressure $p_L$ at which $T_c$ is maximum and explains the quantum critical spin fluctuation origin of the observed Curro $T/T_c$ scaling. It shows that the total spin-lattice relaxation rate will scale with the strength of the heavy electron component $f(T/T^*)$ and hence with $T^*/T^*$ only if quantum critical spin fluctuations determine the intrinsic heavy electron spin-lattice relaxation rate and yields a simple relation between $T^*$ and $T_c$ at $p_L$. It provides the hitherto missing link between the observation of Curro scaling and its physical origin and enables one to identify materials in which quantum critical spin fluctuations provide the pairing glue. Our results support a strategy for finding higher transition temperature heavy electron superconductors that is based on increasing $T^*(p_L)$ [3] and raise the interesting question of whether a similar link can be found for other unconventional superconductors for which Curro scaling has been observed.

Y.Y. is supported by the National 973 Project of China (Grant No. 2015CB921303), the National Natural Science Foundation of China (NSFC Grant No. 11174339), and the Strategic Priority Research Program (B) of the Chinese Academy of Sciences (Grant No. XDB07020200). We thank the Aspen Center for Physics (NSF Grant No. 1066293) for their hospitality during the writing of this paper; Y.Y. thanks the Simons Foundation for its support. N.J.C. thanks NNSA de-na0001842-0 for support.