Dirac semimetal in $\beta$-Cul without surface Fermi arcs

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Edited by Siddharth Parameswaran, Oxford University, Oxford, UK, and accepted by Editorial Board Member Zachary Fisk June 29, 2018 (received for review February 28, 2018)

Anomalous surface states with Fermi arcs are commonly considered to be a fingerprint of Dirac semimetals (DSMs). In contrast to Weyl semimetals, however, Fermi arcs of DSMs are not topologically protected. Using first-principles calculations, we predict that $\beta$-cuprous iodide ($\beta$-Cul) is a peculiar DSM whose surface states form closed Fermi pockets instead of Fermi arcs. In such a fermiological Dirac semimetal, the deformation mechanism from Fermi arcs to Fermi pockets stems from a large cubic term that preserves all crystal symmetries and from the small energy difference between the surface and bulk Dirac points. The cubic term in $\beta$-Cul, usually negligible in prototypical DSMs, becomes relevant because of the particular crystal structure. As such, we establish a concrete material example manifesting the lack of topological protection for surface Fermi arcs in DSMs.

Topological semimetals including Dirac semimetals (DSMs), Weyl semimetals (WSMs), and nodal line semimetals have being attracting enormous attention in contemporary research (1, 2), exhibiting a plethora of exotic phenomena (3–14). In particular, the surface states of such semimetals commonly feature open Fermi arcs rather than closed Fermi pockets (15). The principal existence of Fermi arcs appears robust against potential bulk band hybridizations and has been confirmed by theoretical calculations as well as experimental observations in all type-I and type-II (15, 16) WSM and DSM materials studied so far (17–24).

The topological protection of nondegenerate surface Fermi arcs in WSMs traces back to topological invariants enforcing the connection between Berry flux monopoles of opposite charge, which is realized by pairs of bulk Weyl cones projected to a given surface. In view of DSMs, however, it has been pointed out recently (25, 26) that the doubly degenerate Fermi arcs on side surfaces are not topologically protected and that a cubic term preserving all crystal symmetries can deform Fermi arcs into closed Fermi surfaces, yielding a state we call fermiological DSM. In all DSMs (Na$_3$Bi and Cd$_3$As$_2$) known so far, such a cubic term is negligible, so that doubly degenerate Fermi arcs always appear at the surfaces.

In this work, we predict that $\beta$-cuprous iodide ($\beta$-Cul) (27, 28) is a proposed instance of a fermiological DSM, exhibiting closed Fermi surfaces instead of Fermi arcs on its side surfaces. The band inversion, which can be greatly enhanced with compressive strain along the $c$ axis, happens between the bonding states of Cu-$4s$ orbitals and I-$5p_{x,y,z}$ orbitals. It generates both 3D topological semimetal and 3D topological insulator phases. A crystal symmetry-preserving cubic term, which is usually expected to be negligible in previous DSM materials, is found to be considerably larger because of the unique atomic arrangements in $\beta$-Cul, in sharp contrast to conventional DSMs such as Na$_3$Bi and Cd$_3$As$_2$. In particular, the small energy difference between surface and bulk Dirac points causes a flat surface state along the $\Gamma$–Z direction. In this flat surface state, the cubic term can introduce a gap for $k_z \neq 0$ to deform Fermi arcs into a closed Fermi surface. Our study provides a concrete material example to illustrate the lack of topological protection of surface Fermi arcs in DSMs. The corresponding consequences in angle-resolved photoemission spectroscopy (ARPES) and quantum oscillation measurements are also discussed.

**Crystal Structure**

The crystal chemistry of Cul is characterized by three stable structural phases $\alpha$, $\beta$, and $\gamma$ (27). Here, we focus on the topologically nontrivial properties of the $\beta$ phase. The crystal structure of $\beta$-Cul with the space group $R3m$ is shown in Fig. 1A (27). According to the chemical environment, the I ions can be classified as $I_1$ and $I_2$. $I_1$ is octahedrally coordinated by six Cu atoms, and $I_2$ is coordinated by only two Cu ions parallel to the $c$ axis, resulting in a strong negative crystal field for the $I_2 p$ orbitals and $I_2 p_z$ orbital. As shown in Fig. 1A, the Cu–I$_2$–Cu form tri-layer structures and are connected by $I_2$ ions along the $c$ axis. In the following calculations, we adopt the experimental structural parameters in ref. 27.

**Electronic Structure**

The band structure and density of states (DOS) for $\beta$-Cul are displayed in Fig. 2. Due to the monovalence of Cu, the $d$ orbitals of Cu are fully filled and located at about $-2.5$ eV. The $p$ orbitals of $I_1$ and $p_z$ orbitals of $I_2$ lie far below the Fermi level because of the strongly negative crystal field. Near the Fermi level, the

*Author contributions: C.L., X.W., and J.H. designed research; C.L. and X.W. performed research; C.L., X.W., S.Q., Y.L., R.T., F.-C.Z., and J.H. analyzed data; and C.L., X.W., S.Q., Y.L., R.T., F.-C.Z., and J.H. wrote the paper.

The authors declare no conflict of interest.

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This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1803599115/-/DCSupplemental.
valence and conduction bands are predominantly attributed to the I$_2$-5p$_{xy}$ and Cu-4s orbitals. The most prominent feature in the band structure is that at the Γ point, the Cu-4s band is lower than the I-5p$_{x,y}$ bands by about 0.47 eV and that there is a crossing point along the ΓZ line near the Fermi level, as shown in Fig. 2A. Due to strong spin orbital coupling (SOC) in I ions, we further take SOC into consideration in our calculations. As shown in Fig. 2C, the I-5p$_{x,y}$ bands in the ΓZ line split into $\Lambda_4$ and $\Lambda_0$ ($j_z = \pm \frac{1}{2}$) and $\Lambda_4$ ($j_z = \pm \frac{3}{2}$) bands, and the band inversion at the Γ point is further enhanced to 0.77 eV. As the generalized gradient approximation occasionally tends to underestimate band gaps, we further assert the avenue of band inversion by hybrid functional Heyd–Scuseria–Ernzerhof calculations and also find that the gap can be greatly enhanced through compressive strain along the c axis (SI Appendix, section 2). Furthermore, as the two crossing bands along the ΓZ line belong to different irreducible representations as distinguished by C$_3v$ rotational symmetry about the z axis, this indicates that the 3D Dirac cones near the Fermi level are stable. Notably, the Cu-4s and I-5p$_{x,y}$ [j$_z = \pm \frac{1}{2}$] bands have the same $\Lambda_4$ irreducible representation, which leads to a band anticrossing and a full gap opening around $\sim$0.4 eV. As the parities of Cu-4s and I-5p$_{x,y}$ are opposite at the Γ and the Z point, band inversion will drive the system into a topologically nontrivial phase. Due to the presence of 3D inversion symmetry in β-Cul, we can calculate $Z_2$ topological invariants by analyzing the parity eigenstates at high-symmetry points (29). The parity of the eigenstates near the Fermi level at Γ and Z points is displayed in Fig. 2C. According to our calculations, Cul is a topologically nontrivial semimetal, with 3D $Z_2$ invariants given by (1:000) (SI Appendix, section 1). Furthermore, setting the chemical potential to $\sim$0.4 eV, the system is located in a topological insulator phase with nontrivial $Z_2$ invariants. In total, we thus find that band inversion generates both topological semimetal and topological insulator phases.

Because of bulk–edge correspondence, a topologically nontrivial bulk state is accompanied by gapless surface states. For CuI, these can be obtained by calculating the surface Green function of the seminfinite system through an iterative procedure (30, 31). Fig. 3A shows the edge states on the (001) surface. Interestingly, a surface Dirac cone exists around $\sim$0.4 eV stemming from the nontrivial topological insulator phase, and the corresponding Fermi surface is a closed circle with a left-handed spin texture (SI Appendix, section 6). The surface states of the (100) surface in the conventional cell are shown in Fig. 3B. The energy difference $\Delta$ between the surface Dirac point at Γ and the projections of the bulk Dirac points is extremely small, yielding flat surface states along ΓZ, in sharp contrast to conventional DSMs (6, 15) (SI Appendix, section 7). Despite the band folding along the ΓZ direction (SI Appendix, section 4), we find that the two surface states vanish at the projection of bulk Dirac points and exhibit nonmonotonic dispersion along ΓZ. Furthermore, the lower surface state first sinks below the energy level $E_D$ of the bulk Dirac points, then rises above it, and finally bends down to saturate at it, resulting in three crossing points for $k_z = 0$ at $E_D$ (denoted by white and red circles in Fig. 3B and D). The corresponding Fermi surface of (100) surface at $E_D$ is shown in Fig. 3D. There is one closed nontrivial Fermi pocket centered around $k_z = 0$ and there are two trivial pockets around $k_z = \pi$, which originate from the nontrivial $Z_2$ invariant in the $k_z = 0$ plane and the trivial $Z_2$ invariant in the $k_z = \pi$ plane, respectively. The closed Fermi pocket around $k_z = 0$ does not pass through the projections of the bulk Dirac points (denoted by red circles in Fig. 3D), illustrating that Fermi arcs are absent. Furthermore, the surface states at an exemplary amount of $k_z = \pi/6$ lower than the location $k_{zD}$ of the Dirac point, which exhibit gap opening, are shown in Fig. 3C. We find that the obtained surface states are gapped for all $k_z$ except $k_z = 0$, which plays an important role in deforming Fermi arcs into a closed Fermi surface.

Fig. 1. Crystal structure and primitive Brillouin zone for β-Cul. (A) Crystal structure of β-Cul. Cu–I–Cu trilayers stacking along the c axis are connected by I$_2$ ions. (B) Top view of crystal structure of Cu–I–Cu trilayers. I$_1$ is octahedrally coordinated by six Cu atoms, which generate a negative crystal field. (C) Primitive Brillouin zone for β-Cul.

Fig. 2. Band structures and DOS for β-Cul without SOC and with SOC. (A and B) Band structures and DOS of β-Cul without SOC. The band inversion happens between I$_2$-5p$_{xy}$ and Cu-4s orbitals, and Dirac points are located in ΓZ near the Fermi level. (C and D) Band structures and DOS of β-Cul with SOC. The I-5p$_{x,y}$ bands in the ΓZ line are split, and the band inversion at the Γ point is further enhanced to be 0.77 eV. The orbital weights are represented by the areas of circles and triangles. The particles of the eigenstates and the irreducible representations of bands at the Γ point near the Fermi level are shown. The eigenvalues of C$_3$ for bands along the Γ–Z line are displayed in SI Appendix, section 1.

Fig. 3. Surface states on the (001) surface. (A) Top view of surface states on the (001) surface. (B) Cross section view of surface states on the (001) surface. (C) Surface states at the (001) surface. (D) Surface states at the (001) surface.
Effective Hamiltonian

To characterize the low-energy effective Hamiltonian around the \( \Gamma \) point, which is helpful to understand the origin of the surface Fermi arc break down, we adopt the perspective of the theory of invariants (32). From the band structure, the states around \( \Gamma \) are mainly attributed to I-5p \( s \) and Cu\(-\)4s \( \alpha \) orbitals, and thus these orbitals can be used to construct the basis. Further considering the inversion symmetry in the system, it is convenient to combine these orbitals to form the eigenstates of the inversion symmetry, which are given by

\[
\begin{align*}
|P_{a}^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|I_{a}\rangle \pm |I'_{a}\rangle), \\
|S^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|Cu_{a}\rangle \pm |Cu'_{a}\rangle),
\end{align*}
\]

where the superscript denotes the parity, \( a = p_{z,y}, \) and the I (Cu) \( \alpha \) as well as \( \Gamma' \) (Cu') atoms are related by inversion symmetry. We focus on the low-energy states near the bulk Dirac point. After further taking into account SOC in the atomic picture, we can choose \( |S^{+\pm}\rangle, |P^{+\pm}\rangle, |S^{\mp\pm}\rangle, |P^{\mp\pm}\rangle \) as the basis in \( \mathbf{k} \cdot \mathbf{p} \) theory to construct the effective Hamiltonian around the \( \Gamma \) point. The Hamiltonian to third order in \( \mathbf{k} \) reads

\[
\begin{align*}
H_{\alpha\beta}(\mathbf{k}) &= H_{0} + H_{1} + H_{2} \\
H_{0} &= \epsilon(\mathbf{k}) + M(\mathbf{k})\sigma_{0}\tau_{3} - A(\mathbf{k})\left( k_{x}\sigma_{5}\tau_{2} + k_{y}\sigma_{0}\tau_{1} \right) \\
H_{1} &= \left( D_{2} + D_{3}k_{z}^{2} \right)\left( -k_{x}\sigma_{1}\tau_{2} + k_{y}\sigma_{2}\tau_{2} \right) \\
H_{2} &= -D_{4}k_{z}\left( k_{x}^{2} - k_{y}^{2} \right)\sigma_{1}\tau_{2} + 2k_{x}k_{y}\sigma_{2}\tau_{2},
\end{align*}
\]

where the Pauli matrices \( \sigma \) act in spin and \( \tau \) in orbital space, \( k_{e} = k_{x} \pm ik_{y} \), \( \alpha = C_{0} + C_{1}k_{x}^{2} + C_{2}(k_{x}^{2} + k_{y}^{2}) \), \( M(\mathbf{k}) = M_{0} - M_{1}k_{x}^{2} - M_{2}(k_{x}^{2} + k_{y}^{2}) \), \( A(\mathbf{k}) = A_{0} + A_{1}k_{x}^{2} \), \( D(\mathbf{k}) = ik_{x} + D_{2} + D_{3}k_{z}^{2} \), and \( \tilde{D}(\mathbf{k}) = iD_{4}k_{z}k_{x}^{2} \). The antidiagonal terms contain first-order and third-order terms, which have often been omitted in previous studies, but turn out to be of great importance in \( \beta\)-CuI.

The energy dispersion of the Hamiltonian for the DSM is

\[
E(\mathbf{k}) = \epsilon_{k} \pm \sqrt{M(\mathbf{k})^{2} + 2A(\mathbf{k})k_{z} - \left( D(\mathbf{k}) + \tilde{D}(\mathbf{k}) \right)^{2}},
\]

resulting in two band crossing points \( (0,0, \pm k_{sd}) \) along the \( \Gamma - Z \) line with \( k_{sd} = \sqrt{-\frac{2A_{0}}{M_{0}}} \).

By fitting the bands of the effective model with those of density functional theory (DFT) calculation around the \( \Gamma \) point, the parameters in the effective model are given by \( C_{0} = -0.2070 \) eV, \( C_{1} = 2.0445 \) eV \( \mathbf{Å}^{2} \), \( C_{2} = 12.8481 \) eV \( \mathbf{Å}^{2} \), \( M_{0} = -0.3855 \) eV, \( M_{1} = -6.8288 \) eV \( \mathbf{Å}^{2} \), \( M_{2} = -37.4544 \) eV \( \mathbf{Å}^{2} \), \( A_{0} = 4.0035 \) eV \( \mathbf{Å}^{2} \), \( A_{1} = -1.629.0242 \) eV \( \mathbf{Å}^{2} \), \( D_{2} = 167.799 \) eV \( \mathbf{Å}^{2} \), \( D_{3} = 2.8549 \) eV \( \mathbf{Å}^{2} \), and \( D_{4} = -1.668.6306 \) eV \( \mathbf{Å}^{2} \). In \( \beta\)-CuI, we find that the coefficients in the antidiagonal terms are considerably large and thus cannot be omitted.

In ref. 25, the Fermi arcs on the (100) surface have shown to be not protected by symmetry and can in principle be absent. Still, the effective Hamiltonian \( H_{0} \), up to second order in \( \mathbf{k} \), can give robust surface Fermi arcs. Therefore, \( H_{0} \) must have additional symmetries, which are to some degree artificial and not enforced for DSMs. Consider a pseudo–time-reversal symmetry \( T \) in 2D, which can be defined as \( T = i\sigma_{z}\tau_{z} \cdot K \). Under this operation, the Hamiltonian for \( H(k_{x}, k_{y}, k_{z}) \) at a fixed \( \mathbf{k} \) plane transforms as \( TH(k_{x}, k_{y}, k_{z})T^{-1} = H(-k_{x}, -k_{y}, k_{z}) \). It can be easily verified that the Hamiltonians \( H_{0} \) and \( H_{1} \) are invariant under the operation \( T \). This symmetry, not preserved for the generic realistic system but only for the Hamiltonian \( H_{0}, H_{1} \), and its sides surfaces, can protect gapless surface states for any \( k_{x} < k_{sd} \) planes. The energy difference between the surface Dirac point and the bulk Dirac point is given by (SI Appendix, section 5)

\[
\Delta = \left( \frac{C_{2}}{M_{2}} - \frac{C_{1}}{M_{1}} \right) M_{0}.
\]

The corresponding prototypical surface states on the (100) surface along \( Y - \Gamma - Z \) for \( H_{0} + H_{1} \), with a small \( \Delta \), are shown in Fig. 4A, where the energy of two degenerate flat surface states decreases monotonically with increasing momentum along \( \Gamma Z \). As a consequence of the latter, there are only two points in \( k_{0} = 0 \) on the Fermi surface at \( E_{F} \), that is, the projected bulk Dirac points, and Fermi arcs can robustly appear on the (100) surface. It is, however, the cubic \( H_{2} \) term that breaks this artificial symmetry and naturally introduces gap openings for any \( k_{0} \) except \( k_{0} = 0 \) where fundamental time-reversal symmetry is kept. Taking \( H_{2} \) into consideration, two surface states split, as shown in Fig. 4B, and the prominent feature is that both surface states exhibit a nonmonotonous band dispersion along \( \Gamma Z \), which generates an additional two points in \( k_{0} = 0 \) at \( E_{F} \) in the surface state. As such, the Fermi arcs deform into a closed Fermi pocket, bearing some similarity to a 3D topological insulator surface state. Further increasing coefficients in \( H_{2} \), we find that this will reduce the splitting of surface states along \( \Gamma Z \). Adding either inversion symmetry or time-reversal symmetry breaking, DSMs become WSMs, and Fermi arcs are known to be robust (SI Appendix, section 7). In the presence of both these symmetries, however, the cubic term explicates how Fermi arcs on the surface of a DSM are not topologically protected.

We now turn to a detailed analysis of why despite the above finding, the hallmark of DSM materials discovered previously, such as Na\(_{3}\)Bi and Cd\(_{3}\)As\(_{2}\), still has the appearance of seemingly robust Fermi arcs. We conjecture that this is attributed to a small
the second term includes \( k_s^2 \) and, as such, in comparison with \( H_2 \) has a much weaker effect for small \( k_s \). Therefore, the combined appearance of large \( D_z \) as well as small \( \Delta \) in \( \beta \)-CuI triggers the breakdown of surface Fermi arcs.

**Discussion**

We elaborate on experimental evidence derived from the breakdown of surface Fermi arcs due to a significant cubic term. First, aiming at the effect of the cubic term in the bulk, the in-plane energy dispersion for a specific \( k_z \) is
\[
E(k_z) = \varepsilon_0 \pm \sqrt{f_1 + f_2 |k|^2 + f_3 |k|^4}.
\]
If \( D_z \) is large, the coefficient \( f_3 = M^2_z + D^2_z k^2_z \) should exhibit noticeable \( k_z \) dependence, which could be identified upon fitting the band structure against ARPES measurements. Second, as the splitting of (100) surface states along \( \Gamma-Z \) is directly related to the cubic term, this splitting can likewise be obtained in ARPES and is expected to be relatively large as well as strongly \( k_z \) dependent. In addition, the change of nature of the surface states from arcs to closed Fermi pockets hints at immediate experimental implications. First, terminating Fermi arcs and closed Fermi surfaces exhibit qualitative differentiable shape differences in ARPES measurements. For the former, when two Fermi arcs meet at the projection of bulk Dirac points, there is a singular change in slope, whereas for the latter, the closed Fermi surface has a smooth curvature everywhere and does not pass through the projections of bulk Dirac points. Second, the distinct behavior of surface Fermi arcs vs. closed surface Fermi pockets in quantum oscillation measurements can be used to contrast them. In the former case, the quantum oscillation frequency \( F_z \) is strongly dependent on the sample thickness due to the Weyl orbits (33, 34). In triangle-shaped samples, quantum oscillations can be even unobservable in experiment (34). In the latter case, fermions acquire a measurable Berry phase of \( \pi \) as they encircle the Fermi contour, similar to topological insulators. In contrast to the former case, quantum oscillations can exist in triangle-shaped samples (35–37) and exhibit weak thickness dependence.

**Conclusion**

Based on first-principles calculations we predict that \( \beta \)-CuI is a topological unconventional DSM exhibiting closed Fermi surfaces instead of Fermi arcs on its side surfaces. The theoretical discovery of \( \beta \)-CuI provides explicit proof that the Fermi arcs in DSMs are not topologically protected. Our study also suggests that halide compounds can be a fertile ground to explore novel topological properties.

**Methods**

Our calculations are performed using DFT as implemented in the Vienna ab initio simulation package (VASP) code (38–40). The Perdew–Burke–Ernzerhof (PBE) exchange-correlation functional and the projector-augmented-wave (PAW) approach are used. Throughout this work, the cutoff energy is set to 500 eV for expanding the wave functions into plane-wave basis. In the calculation, the Brillouin zone is sampled in the \( k \) space within the Monkhorst–Pack scheme (41). On the basis of the equilibrium structure, the \( k \) mesh used is \( 6 \times 6 \times 6 \) and \( 10 \times 10 \times 2 \) for primitive and conventional cells, respectively.

ACKNOWLEDGMENTS. We thank Chen Fang and Lunhui Hu for helpful discussion. This work is supported by the Ministry of Science and Technology of China 973 program (Grants 2015CB921300 and 2017YFA0303100), the National Science Foundation of China (Grant NSFC-11334012), and the Strategic Priority Research Program of Chinese Academy of Sciences (Grant XD-B07000000). This work is supported in part by the Key Research Program of the Chinese Academy of Sciences (Grant XDPB08-4), the Strategic Priority Research Program of Chinese Academy of Sciences (Grant XDB28000000), and the National Science Foundation of China (Grant NSFC-11674278). The work in Würzburg is supported by the European Research Council (ERC) through ERC-SE-SG-TOPOELECTRICS-336012 and by the German Research Foundation (DFG) through the special research unit (SFB) DFG-SFB 1170 (Project B04) and through the priority programme (SPP) DFG-SPP 1666.


