Non-Hermitian Skin Modes Induced by On-Site Dissipations and Chiral Tunneling Effect

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In this Letter, we study the conditions under which on-site dissipations can induce non-Hermitian skin modes in non-Hermitian systems. When the original Hermitian Hamiltonian has spinless time-reversal symmetry, it is impossible to have skin modes; on the other hand, if the Hermitian Hamiltonian has spinful time-reversal symmetry, skin modes can be induced by on-site dissipations under certain circumstances. As a concrete example, we employ the Rice-Mele model to illustrate our results. Furthermore, we predict that the skin modes can be detected by the chiral tunneling effect; that is, the tunneling favors the direction where the skin modes are localized. Our Letter reveals a no-go theorem for the emergence of skin modes and paves the way for searching for quantum systems with skin modes and studying their novel physical responses.

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Introduction.—Non-Hermitian Hamiltonians [1–3], which describe the nonconservative phenomena [4], have been widely studied recently [4–31]. It has been shown that some non-Hermitian Hamiltonians with open boundary condition can never be characterized by Bloch Hamiltonians [31–60]. To be more precise, the open boundary spectra may collapse compared to the periodic boundary spectra, along with the emergence of non-Hermitian skin modes [32]. It has been shown that both phenomena can be well understood with the concept of the generalized Brillouin zone (GBZ) [32–37], which is a generalization of the Brillouin zone (BZ) defined in systems (Hermitian or non-Hermitian) with open boundaries. When the GBZ coincides with the BZ, the open boundary spectra can be described by the Bloch Hamiltonian with no skin modes, and the conventional bulk-boundary correspondence still holds. On the other hand, if GBZ is distinct from BZ, the open boundary spectra collapses, and skin modes along with the anomalous bulk-boundary correspondence emerge at the same time [32]. Inspired by the theoretical proposal, non-Hermitian skin modes have been observed experimentally in the classical wave systems recently [61–64]. Finding the conditions for the emergence of skin modes in quantum systems and investigating the corresponding novel physical responses are interesting and challenging [65–98].

On-site dissipations are well-controlled non-Hermitian terms that can be realized experimentally in both classical and quantum systems [4–8,99–105]. In contrast to the nonreciprocal terms, like $\sum_i(t\hat{a}_i^\dagger\hat{b}_i+2t\hat{b}_i^\dagger\hat{a}_i)$, on-site dissipations, such as $\sum_i(i\gamma_a\hat{a}_i^\dagger\hat{a}_i+i\gamma_b\hat{b}_i^\dagger\hat{b}_i)$, do not favor any special hopping direction. Although it has been revealed that skin modes can be induced by on-site dissipations [13,31,41,63], their exact relation is still unclear.

In this Letter, we show that, if a Hermitian nonsuperconducting system has spinless time-reversal symmetry (TRS), on-site dissipations will not induce non-Hermitian skin modes. However, if the Hermitian system has spinful TRS, it is possible for the system to have skin modes, depending on whether the system has inversion symmetry (IS) and its representation. As a concrete example, we use the Rice-Mele model [106] to illustrate our results. The novel physical responses of skin modes are also investigated.

Non-Hermitian Hamiltonians with on-site dissipations.—We start from the following one-dimensional (1D) *Hermitian* Hamiltonian,

$$\hat{H}=\hat{H}_s+\hat{H}_b+\hat{H}_{s-b}. \tag{1}$$

Here $\hat{H}_s=\sum_{i,j}\sum_{\mu,\nu}t_{ij}^{\mu\nu}\hat{c}_{i\mu}^\dagger\hat{c}_{j\nu}$ is the system Hamiltonian with which we are concerned, where i, j and μ, ν label lattice sites and band (or spin) indexes, respectively; $\hat{H}_b=\sum_{i,p_\mu,\mu}(\varepsilon_{p_\mu}-\mu_{p_\mu})\hat{b}_{ip_\mu}^\dagger\hat{b}_{ip_\mu}$ comes from a free Fermion bath, where p_μ is the internal degrees of the bath, and $\hat{H}_{s-b}=\sum_{i,p_\mu,\mu}V_{p_\mu\mu}(\hat{c}_{i\mu}^\dagger\hat{b}_{ip_\mu}+\hat{b}_{ip_\mu}^\dagger\hat{c}_{i\mu})$ is the system-bath coupling term. We first focus on the periodic boundary condition. In the Supplemental Material, Sec. I, we show that the following non-Hermitian effective Hamiltonian can be obtained by using the standard Green's function method [107]:

$$\mathcal{H}_{s,\mathrm{eff}}(k)=\mathcal{H}_s(k)-i\gamma\Gamma_0,\qquad \mathcal{H}_s(k)=\mathcal{H}_s^\dagger(k), \tag{2}$$

where $[\mathcal{H}_s(k)]_{\mu\nu} = \sum_{l_{\mu\nu}} t^{\mu\nu}_{l_{\mu\nu}} e^{ikl_{\mu\nu}}$ is the Bloch Hamiltonian of the system, γ is proportional to the density of states (DOS) of the external bath and the system-bath coupling strength, and Γ_0 is a diagonal matrix, whose matrix elements represent the dissipations for each band (or spin). This kind of dissipation is dubbed "on-site dissipation" in this Letter. Exploring the condition for the emergence of skin modes in Eq. (2) is the central topic of this Letter. The extension to the general non-Hermitian Hamiltonians will be discussed in the final section.

Non-Hermitian symmetries and skin modes.—The main results of this Letter can be summarized as follows. If $\mathcal{H}_s(k)$ in Eq. (2) preserves TRS but breaks particle-hole symmetry (PHS) [108], then, (i) for the spinless case, it is impossible to have skin modes. (ii) For the spinful case, if the skin modes are to emerge, one of the following three conditions must be satisfied: (a) $\mathcal{H}_s(k)$ breaks IS; (b) $\mathcal{H}_s(k)$ preserves IS represented by $\mathcal{P}$, but $\{\mathcal{P}, \Gamma_0\} = 0$; or (c) $\mathcal{H}_s(k)$ preserves IS and $[\mathcal{P}, \Gamma_0] = 0$, but $\{\mathcal{P}, \mathcal{T}\} = [\Gamma_0, \mathcal{T}] = 0$, where $\mathcal{T}$ represents TRS. Whereas, if Γ_0 and the symmetries of $\mathcal{H}_s(k)$ do not satisfy the above three conditions, it is impossible to have skin modes. Our results reveal a no-go theorem for the emergence of skin modes. For example, if $\mathcal{T} = i\sigma_y \mathcal{K}^*$, where $\mathcal{K}^*$ represents the complex conjugate operator $\mathcal{P} = \tau_z$, it is impossible to have skin modes in Eq. (2) [109]. However, if $\mathcal{T} = i\sigma_y \mathcal{K}^*$, $\mathcal{P} = \tau_x$, skin modes can be induced by the on-site dissipation $\Gamma_0 = \tau_z$ due to $\{\mathcal{P}, \Gamma_0\} = 0$.

Our derivation of the main results is based on the GBZ theory [32,35–37]. Here we briefly summarize the procedure of our derivation. All the details can be found in the Supplemental Material [110]. We first write down all the non-Hermitian symmetry groups that Eq. (2) belongs to when $\mathcal{H}_s(k)$ preserves TRS but breaks PHS. After that, we use the GBZ theory to derive which non-Hermitian symmetry groups forbid the emergence of skin modes. As shown in Table I, all the elements of 1D Hermitian (non-Hermitian) symmetry groups are listed in the first (second and fifth) row. Here $\mathcal{T}$, $\mathcal{C}$, $\mathcal{P}$, $\bar{\mathcal{T}}$, $\bar{\mathcal{C}}$ represent TRS, PHS, IS, anomalous time-reversal symmetry (TRS†), and anomalous particle-hole symmetry (PHS†), respectively [71,73], while the others represent the combination of the above five symmetries; e.g., $\mathcal{PT}$ represents the combination of TRS and IS [111]. The symmetry constraints to the Bloch Hamiltonian are summarized in the third and sixth rows of Table I [73]. We note that the derivation of skin modes requires the information of the GBZ Hamiltonian, which is an extension of the Bloch Hamiltonian to the entire complex plane via a substitution, $\mathcal{H}(k) \to \mathcal{H}(\beta = e^{ik})$ where $k \in \mathbb{C}$ [32,35,37]. In the Supplemental Material, Sec. II, we show how symmetries constrain the characteristic equation $f(\beta, E) = \det[E - \mathcal{H}(\beta)]$, and the result can be summarized in the fourth and seventh rows of Table I.

In Eq. (2), an important observation is that all the non-Hermitian symmetries of $\mathcal{H}_{s,\mathrm{eff}}(k)$ have a Hermitian origin. For example, when $\mathcal{H}_s(k)$ preserves TRS represented by $U_{\mathcal{T}}\mathcal{K}^*$, it automatically preserves TRS† due to $\mathcal{H}_s(k) = \mathcal{H}_s^\dagger(k)$. It can be deduced that, if $[\Gamma_0, U_{\mathcal{T}}] = 0$, then TRS is broken but TRS† is preserved for the overall non-Hermitian Hamiltonian $\mathcal{H}_{s,\mathrm{eff}}(k)$. On the other hand, if $\{\Gamma_0, U_{\mathcal{T}}\} = 0$, TRS† is broken but TRS is preserved. This phenomena is called symmetry ramification [73]. In the Supplemental Material, Sec. III, we show that the ramification for other symmetries obeys a similar rule as shown in Table I. Finally, if $\mathcal{H}_s(k)$ preserves TRS but breaks PHS, in the Supplemental Material, Sec. III, we show that $\mathcal{H}_{s,\mathrm{eff}}(k)$ belongs to the following non-Hermitian symmetry groups: $G_{\mathcal{T}_\pm}$, $G_{\mathcal{T}_\pm,(\mathcal{PT})_\pm}$, $G_{\mathcal{T}_\pm,(\mathcal{PC})_\pm}$, $G_{\mathcal{T}_\pm,(\mathcal{P}\bar{\mathcal{T}})_\pm}$, $G_{\mathcal{T}_\pm,(\mathcal{P}\bar{\mathcal{C}})_\pm}$, $G_{\bar{\mathcal{T}}_\pm}$, $G_{\bar{\mathcal{T}}_\pm,(\mathcal{PT})_\pm}$, $G_{\bar{\mathcal{T}}_\pm,(\mathcal{PC})_\pm}$, $G_{\bar{\mathcal{T}}_\pm,(\mathcal{P}\bar{\mathcal{T}})_\pm}$, and $G_{\bar{\mathcal{T}}_\pm,(\mathcal{P}\bar{\mathcal{C}})_\pm}$. Here G represents the group generators. For example, $G_{\mathcal{T}_-,(\mathcal{PT})_+} = \{I(\text{identity element}), \mathcal{P} = U_{\mathcal{P}}, \mathcal{T}_- = U_{\mathcal{T}_-}\mathcal{K}^*, (\mathcal{PT})_+ = U_{(\mathcal{PT})_+}\mathcal{K}^*\}$, with $U_{\mathcal{T}_-}U^*_{\mathcal{T}_-} = -1$ and $U_{(\mathcal{PT})_+}U^*_{(\mathcal{PT})_+} = 1$.

For *all* the symmetry groups listed above, skin modes are absent when G contains spinless TRS† ($\bar{\mathcal{T}}_+$) or IS ($\mathcal{P}$) or both (see Supplemental Material, Sec. IV for details). An exceptional case is $G_{\bar{\mathcal{T}}_-,(\mathcal{P}\bar{\mathcal{T}})_+}$, in which skin modes emerge with the presence of IS. Note that this result can be generalized to apply to non-Hermitian Hamiltonians of any form. The derivation is based on the constraints that

TABLE I. Non-Hermitian symmetry ramifications of Eq. (2). All the elements of 1D Hermitian (non-Hermitian) symmetry groups are listed in the first (second and fifth) row. If $\mathcal{H}_s(k)$ has one of the eight Hermitian symmetries listed in the first row, then depending on the commutation relation, the corresponding non-Hermitian symmetries, listed in the second and fifth rows, will be preserved for $\mathcal{H}_{s,\mathrm{eff}}(k)$. The third and sixth rows represent the symmetry constraints to the characteristic equation $f(\beta, E) = \det[E - \mathcal{H}(\beta)]$, where $\mathcal{H}(\beta)$ is the non-Bloch Hamiltonian with $\beta = e^{ik}$ and $k \in \mathbb{C}$. In the Supplemental Material, Sec. III D, we use an example to illustrate the application of Table I.

	Hermitian	I	$\mathcal{PT}$	$\mathcal{P}$	$\mathcal{T}$	$\mathcal{TC}$	$\mathcal{PC}$	$\mathcal{PTC}$	$\mathcal{C}$
$[\Gamma_0, U_X] = 0$	Non-Hermitian	I	$\mathcal{P}\bar{\mathcal{T}}$	$\mathcal{P}$	$\bar{\mathcal{T}}$	$\mathcal{TC}$, $\bar{\mathcal{T}}\bar{\mathcal{C}}$	$\mathcal{P}\bar{\mathcal{C}}$	$\mathcal{PTC}$, $\mathcal{P}\bar{\mathcal{T}}\bar{\mathcal{C}}$	$\bar{\mathcal{C}}$
	$U_X^{-1}\mathcal{H}(k)U_X =$	$\mathcal{H}(k)$	$\mathcal{H}^t(k)$	$\mathcal{H}(-k)$	$\mathcal{H}^t(-k)$	$-\mathcal{H}^\dagger(k)$	$-\mathcal{H}^*(k)$	$-\mathcal{H}^\dagger(-k)$	$-\mathcal{H}^*(-k)$
	$f(\beta, E) =$	$f(\beta, E)$		$f(1/\beta, E)$		$f(1/\beta^*, -E^*)$		$f(\beta^*, -E^*)$	
$\{\Gamma_0, U_X\} = 0$	Non-Hermitian	$\mathcal{T}\bar{\mathcal{T}}$, $\mathcal{C}\bar{\mathcal{C}}$	$\mathcal{PT}$	$\mathcal{PT}\bar{\mathcal{T}}$, $\mathcal{PC}\bar{\mathcal{C}}$	$\mathcal{T}$	$\bar{\mathcal{T}}\mathcal{C}$, $\mathcal{T}\bar{\mathcal{C}}$	$\mathcal{PC}$	$\mathcal{P}\bar{\mathcal{T}}\mathcal{C}$, $\mathcal{PT}\bar{\mathcal{C}}$	$\mathcal{C}$
	$U_X^{-1}\mathcal{H}(k)U_X =$	$\mathcal{H}^\dagger(k)$	$\mathcal{H}^*(k)$	$\mathcal{H}^\dagger(-k)$	$\mathcal{H}^*(-k)$	$-\mathcal{H}(k)$	$-\mathcal{H}^t(k)$	$-\mathcal{H}(-k)$	$-\mathcal{H}^t(-k)$
	$f(\beta, E) =$	$f(1/\beta^*, E^*)$		$f(\beta^*, E^*)$		$f(\beta, -E)$		$f(1/\beta, -E)$	

symmetries impose on the characteristic equation (shown in the fourth and seventh rows of Table I) [110] and the GBZ condition (shown in the Supplemental Material, Sec. IV) [32,35–37,112,113]. For example, if the non-Hermitian system has and only has TRS†, according to the fourth row of Table I, the characteristic equation satisfies $f(\beta, E) = f(1/\beta, E)$. The GBZ conditions for the systems with spinless ($\bar{\mathcal{T}}_+$) case and spinful ($\bar{\mathcal{T}}_-$) case are $|\beta_p(E)| = |\beta_{p+1}(E)|$ and $|\beta_{p-1}(E)| = |\beta_p(E)|$ & $|\beta_{p+1}(E)| = |\beta_{p+2}(E)|$, respectively, where β_i is the ith largest root (order by absolute value) of $f(\beta, E) = 0$, and p is the order of the pole of $f(\beta, E) = 0$. Therefore, $\bar{\mathcal{T}}_+$ forbids the emergence of skin modes [114], while $\bar{\mathcal{T}}_-$ does not [115]. In the Supplemental Material, Sec. IV, we also provide numerical verifications for all the symmetry groups with which we are concerned, which are consistent with our derivations.

Now we show that the conclusions in the last paragraph are equivalent to the main results discussed at the beginning of this section. When $\mathcal{H}_s(k)$ has spinless TRS with $\mathcal{T}_+ = \mathcal{K}^*$, $\mathcal{H}_{s,\mathrm{eff}}(k)$ must preserve spinless TRS† with $\bar{\mathcal{T}}_+ = \mathcal{K}^t$ due to $[U_{\mathcal{T}_+}, \Gamma_0] = 0$. Thus, it is impossible to have skin modes. This is the main result (i) discussed above. For the spinful case with $\mathcal{T}_- = U_{\mathcal{T}_-}\mathcal{K}^*$, if $\mathcal{H}_s(k)$ has $\mathcal{P} = U_\mathcal{P}$ and $[U_\mathcal{P}, \Gamma_0] = 0$, the non-Hermitian Hamiltonian must preserve IS. This forbids the emergence of skin modes in general. For the exceptional case, the existence of $\bar{\mathcal{T}}_-$ and $(\mathcal{P}\bar{\mathcal{T}})_+$ symmetries implies the Hermitian Hamiltonian $\mathcal{H}_s(k)$ must preserve $\mathcal{T}_-$ and $(\mathcal{P}\mathcal{T})_+$ symmetries, which ultimately leads to $\{\mathcal{P}, \mathcal{T}\} = 0$ [110,116–118], and this is equivalent to the main result (ii).

Example.—In order to verify our results, we use the Rice-Mele model as an example,

$$\mathcal{H}_{\mathrm{RM}}(k) = (t_1 + t_2 \cos k)\sigma_x + t_2 \sin k\sigma_y + \mu\sigma_z, \quad (3)$$

which preserves $\mathcal{T}_+ = \mathcal{K}^*$, $(\mathcal{P}\mathcal{C})_- = \sigma_y\mathcal{K}^t$, $\mathcal{P}\mathcal{C}\mathcal{T} = \sigma_y\mathcal{K}^\dagger$. Since Eq. (3) preserves $\mathcal{T}_+$ and breaks $\mathcal{P}$, in order to induce skin modes with on-site dissipations, we can either break TRS or add spin-orbit coupling (see Supplemental Material, Sec. V for details). For the spinless case, as shown in Fig. 1(a), we study the case where the Rice-Mele model breaks TRS,

$$\mathcal{H}_{\mathrm{spinless}}(k) = \mathcal{H}_{\mathrm{RM}}(k) + \lambda \sin k\sigma_z + i\gamma\sigma_z, \quad (4)$$

where λ controls the term that breaks TRS. It is easy to verify that only $(\mathcal{P}\mathcal{C})_-$ symmetry is preserved for Eq. (4), which implies $f(\beta, E) = f(\beta, -E)$. According to the GBZ condition $|\beta_p| = |\beta_{p+1}|$ shown in the Supplemental Material, Sec. IV, we can deduce that (i) the spectrum is formed by pairs as $(E, -E)$, and (ii) the roots of the characteristic equation satisfy $\beta(E) = \beta(-E)$, which means the sub-GBZs [37] for the E and $-E$ bands are the same. All the wave functions of Eq. (4) with $t_1 = \lambda = 2$, $t_2 = \mu = \gamma = 1$, and

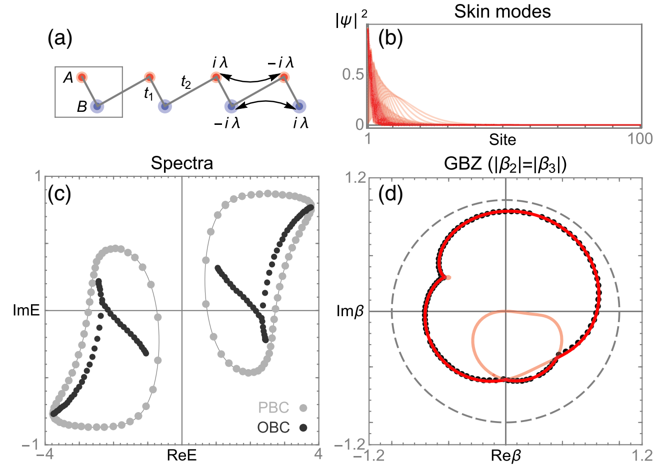

FIG. 1. Skin modes induced by the on-site dissipation in the TRS breaking Rice-Mele model, i.e., Eq. (4). (a) Schematic diagram of the Hermitian part, namely, $\mathcal{H}_{\mathrm{RM}}(k) + \lambda \sin k\sigma_z$. (b)–(d) All the eigenstates (skin modes), open-periodic boundary condition spectrum, and numerical result of GBZ (black points) and auxiliary GBZ [37] (red lines) of the system, respectively. Here the PBC and OBC in (c) are the abbreviations for the periodic boundary condition and open boundary condition, respectively.

$N = 100$ (lattice site) are plotted in Fig. 1(b); one can notice that they all localize at the left boundary. The discrepancy between the periodic and open boundary spectrum, as shown in Fig. 1(c) [119], also reveals the existence of skin modes [32,36,120,121]. The corresponding numerical calculation of GBZ (black points) and auxiliary GBZ [37] (red lines) are shown in Fig. 1(d), which are both inside the unit circle (gray dashed lines). In the Supplemental Material, Sec. V, we show that, regardless of the value of μ, the skin modes exist when $\lambda t_1 t_2 \neq 0$. This means the on-site dissipations can also induce skin modes in the Su-Schrieffer-Heeger model [122] when TRS is broken.

For the spinful case, since the Rice-Mele model breaks IS, on-site dissipation can induce skin modes if we add spin-orbit coupling. Therefore, the following Bloch Hamiltonian with intrinsic and shortest ranged Rashiba spin-orbit coupling [123] is studied:

$$\mathcal{H}_{\mathrm{spinful}}(k) = \mathcal{H}_{\mathrm{RM}}(k)s_0 + \mathcal{H}_{\mathrm{soc}}(k) + i\gamma\sigma_z s_0,$$
$$\mathcal{H}_{\mathrm{soc}}(k) = \lambda_I \sin k\sigma_z s_z - \lambda_R\sigma_y(s_x - \sqrt{3}s_y)/2, \quad (5)$$

where s is the spin Pauli matrix. Under the action of spinful TRS†, $|\beta, E, \uparrow\rangle$ maps to $|1/\beta, E, \downarrow\rangle$. Therefore, a left localized eigenstate with $|\beta| < 1$ will be mapped to the right one with $|\beta| > 1$. These skin modes are called Z_2 skin modes [38] and protected by TRS†. Indeed, according to the GBZ condition, we require $|\beta_{p-1}| = |\beta_p| = 1/r_0$ for one spin band and $|\beta_{p+1}| = |\beta_{p+2}| = r_0$ for the other. The absence of IS implies there is no guarantee for $1/r_0 = r_0$. Therefore, skin modes can emerge. This can be checked by the comparison of open-periodic boundary spectrum and

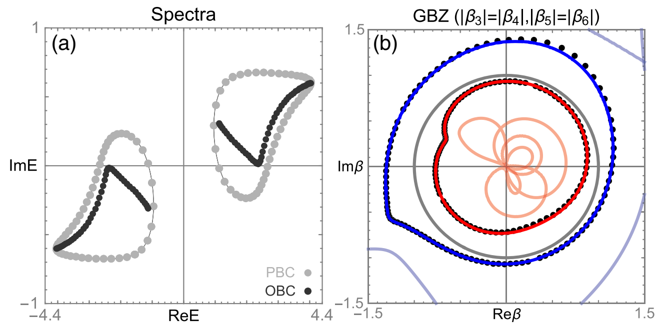

FIG. 2. Z_2 skin modes induced by the on-site dissipation in the Rice-Mele model with spin-orbit coupling, i.e., Eq. (6). (a),(b) The periodic-open boundary condition spectrum and the corresponding auxiliary GBZ (solid lines) and numerical calculated GBZ (Black points) of the system, respectively. Notice that the GBZ condition for the system with spinful TRS† is $|\beta_{p-1}| = |\beta_p|$ & $|\beta_{p+1}| = |\beta_{p+2}|$, where $p = 4$ in our model.

the corresponding GBZ shown in Fig. 2 with the following parameters: $t_1 = \lambda_I = 2$, $t_2 = \mu = \lambda_R = \gamma = 1$, and $N = 50$. As shown in Fig. 2(b), the GBZ for one spin band (the red lines containing the black points) is larger than 1, and the other (the blue lines containing the black points) is smaller than 1. In the Supplemental Material, Sec. VII, we provided a *Mathematica* code to calculate the corresponding GBZ and auxiliary GBZ [110].

Chiral tunneling effect.—When looking for a proper physical observable for detecting skin modes, the local DOS (LDOS) may be the first physical quantity that comes to mind. However, we found that, even if the skin modes are localized at one boundary, say, the left boundary as shown in Fig. 3(a), it will not make the left LDOS much larger than the right one. As shown in Fig. 3(b), we plot the LDOS at each boundary of the Bloch Hamiltonian $\mathcal{H}_{\text{spinless}}(k) - i\gamma\sigma_0$ with open boundary condition (labeled by H_{OBC}), where

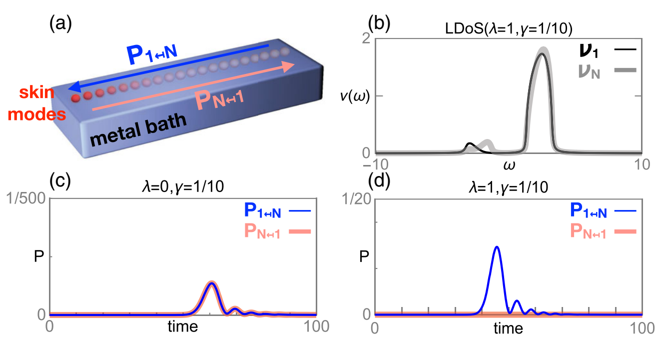

FIG. 3. LDOS and chiral tunneling effect induced by the skin modes. (a) The setup and corresponding left localized skin modes in the model $\mathcal{H}_{\text{spinless}}(k) - i\gamma\sigma_0$ with open boundary condition, where $t_1 = 2$, $t_2 = \mu = 1$, $\gamma = 1/10$, and $N = 50$. (b) The LDOS at each boundary. There is no huge difference between them. (c), (d) The tunneling without and with skin modes, respectively. With the increasing of λ in Eq. (4), skin modes emerge and the tunneling becomes chiral.

the black line (ν_1) and gray line (ν_N) represent the left and right LDOS, respectively. There is no huge difference between them. In the Supplemental Material, Sec. VI, we show that the LDOS at site i can be expressed as

$$\nu_i(\omega) = -\frac{1}{\pi}\sum_n \text{Im}\left[\frac{\langle i|\beta_n^R\rangle\langle\beta_n^L|i\rangle}{\omega - E_n}\right], \tag{6}$$

where $H_{\text{OBC}}|\beta_n^R\rangle = E_n|\beta_n^R\rangle$, $H_{\text{OBC}}^\dagger|\beta_n^L\rangle = E_n^*|\beta_n^L\rangle$, and $\langle\beta_m^L|\beta_n^R\rangle = \delta_{mn}$ [124]. We note that, in the thermodynamic limit, $|\beta_n^R\rangle$ is a superposition of two non-Bloch waves with the same $|\beta_n| = r_n$ [35,37]. It can be further shown that $\langle i|\beta_n^R\rangle \propto r_n^i$ and $\langle\beta_n^L|i\rangle \propto 1/r_n^i$ [110]. Therefore, the contribution of skin modes in Eq. (6) cancels, which explains the numerical results of Fig. 3(b). Consequently, the LDOS is ineffective for detecting skin modes.

We now show that the existence of skin modes can be detected by the chiral tunneling effect due to the unidirectional nature of the non-Hermitian skin effects. This can be intuitively expected, as the model with skin effect can be related to a model with nonreciprocal terms (which implies an asymmetric tunneling) by applying a proper basis (or gauge) transformation [17]. As shown in Figs. 3(c) and 3(d), we plot $P_{N\leftarrow 1}(t)$ and $P_{1\leftarrow N}(t)$ of H_{OBC} for different values of λ, where $P_{f\leftarrow i}(t) = |\langle f|U(t)|i\rangle|^2 = |\langle f|e^{-i\hat{H}t}|i\rangle|^2$ is the tunneling strength from site i to site j. When the TRS-breaking parameter λ increases from zero to a nonzero value, skin modes emerge; in the meantime, $P_{1\leftarrow N}(t)$ increases and $P_{N\leftarrow 1}(t)$ decreases. This means the tunneling along the direction in which the skin modes are localized is favored. Based on the non-Bloch theory, in the Supplemental Material, Sec. VI, we show that $P_{N\leftarrow 1}(t) = |\langle N|U(t)|1\rangle|^2 \propto r_n^{N-1}$ and $P_{1\leftarrow N}(t) = |\langle 1|U(t)|N\rangle|^2 \propto r_n^{1-N}$, where r_n represents the localization length of the skin mode $|\beta_n^R\rangle$. This means the strength of the asymmetric tunneling exponentially depends on the localization length of skin modes [125]. We finally note that the chiral tunneling effect in our model can be experimental controlled by tuning the external magnetic field, which may be useful in the electronics studies.

Discussions and conclusions.—Our results can be applied to more general non-Hermitian Hamiltonians. For example, suppose that the on-site dissipation is a function of k, that is, $\mathcal{H}_{s,\text{eff}}(k) = \mathcal{H}_s(k) - i\gamma(k)\Gamma_0$; the results of Table I remain valid if $\gamma(k)$ is an even function. Whereas if $\gamma(k)$ is an odd function, the results between commutative and anticommutative relations in Table I will be interchanged. For the general case $\mathcal{H}_{s,\text{eff}}(k) = \mathcal{H}_s(k) + \Sigma(k)$, where $\Sigma(k) = \Sigma(\omega = 0, k)$ is the self-energy correction at zero frequency [20,21], once the Hamiltonian has spinless TRS† ($\bar{\mathcal{T}}_+$) or IS ($\mathcal{P}$) or both, skin modes are absent except in the case $G_{\bar{\mathcal{T}}_-,(\mathcal{P}\bar{\mathcal{T}})_+}$ [126].

In summary, our results provide a new approach to realize and control skin modes by tuning the Hermitian Hamiltonian. On the theoretical side, our standard Green's function method paves the way for the study of the novel physical responses induced by non-Hermitian skin modes. On the experimental side, we expect our models and the prediction of the chiral tunneling effect can be realized and observed in various physical systems.

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[108] This means the Hermitian system $\mathcal{H}_s(k)$ is nonsuperconducting.
[109] This is because Γ_0 is a diagonal matrix. As a result, we have $[\mathcal{P}, \Gamma_0] = [\mathcal{P}, \mathcal{T}] = 0$, which breaks the three conditions of the spinful case in our main results.
[110] See Supplemental Material athttp://link.aps.org/supplemental/10.1103/PhysRevLett.125.186802 for more details, which includes (i) Hermitian and non-Hermitian symmetries, (ii) symmetries and skin modes, (iii) Non-Hermitian Rice-Mele model, (iv) Chiral tunneling effect, and (v) the *Mathematica* code.
[111] We note that if a system has $\mathcal{PT}$ symmetry, this does not imply the system has TRS or IS.
[112] Z. Yang (to be published).
[113] The GBZ condition, for example $|\beta_p(E)| = |\beta_{p+1}(E)|$, tells us that, if for some $E_0 \in \mathbb{C}$, $|\beta_p(E_0)| = |\beta_{p+1}(E_0)|$, then E_0 is an asymptotic eigenvalue of the Hamiltonian with open boundary condition, where β_i is the ith largest (ordered by the absolute values) root of $f(\beta, E) = 0$ and p is the order of the pole of $f(\beta, E)$.

[114] This is because, for any given $E_0 \in \mathbb{C}$, the symmetry constraints to the characteristic equation $f(\beta, E_0) = f(1/\beta, E_0)$ implies the roots must come in pairs as $(\beta, 1/\beta)$. Combining this with the GBZ condition $|\beta_p(E_0)| = |\beta_{p+1}(E_0)|$, we can have $|\beta_p| = |\beta_{p+1}| = 1$, which indicates the absence of skin modes by recalling $\beta = e^{ik}$.

[115] This is because the GBZ condition is changed from $|\beta_p(E)| = |\beta_{p+1}(E)|$ to $|\beta_{p-1}(E)| = |\beta_p(E)|$ & $|\beta_{p+1}(E)| = |\beta_{p+2}(E)|$. As a result, the symmetry constraint $f(\beta, E) = f(1/\beta, E)$ can no longer forbid the emergence of skin modes.

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[125] When $r_n < 1$, it seems that the term r_n^{1-N} will tend to infinity in the limit of $N \to \infty$. However, in the Supplemental Material, Sec. VI, we show that the tunneling probability is bounded to less than 1, when the system is purely dissipative.

[126] We note that this conclusion can only be applied to the cases in which $\mathcal{H}_{s,\mathrm{eff}}(k)$ has neither PHS nor PHS†.