



ScQ cloud quantum computation for generating Greenberger-Horne-Zeilinger states of up to 10 qubits

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In this study, we introduce an online public quantum computation platform, named as ScQ, based on a 1D array of a 10-qubit superconducting processor. Single-qubit rotation gates can be performed on each qubit. Controlled-NOT gates between nearest-neighbor sites on the 1D array of 10 qubits are available. We show the online preparation and verification of Greenberger-Horne-Zeilinger states of up to 10 qubits through this platform for all possible blocks of qubits in the chain. The graphical user interface and quantum assembly language methods are presented to achieve the above tasks, which rely on a parameter scanning feature implemented on ScQ. The performance of this quantum computation platform, such as fidelities of logic gates and details of the superconducting device, is presented.

quantum computation, quantum information, quantum entanglement

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1 Introduction

Quantum computers aim to realize quantum algorithms that may outperform classical computers. Recently, quantum supremacy and quantum advantage have been successfully demonstrated for the sampling output of random quantum

circuits and Gaussian boson in laboratories [1-3]. Moreover, cloud (online) quantum computation (CQC) can be accessed worldwide and conveniently used for various aims, including research, application exploration and education. Some CQC platforms based on superconducting processors have been launched for use online, such as the IBM quantum experience (<https://quantum-computing.ibm.com>), on which a series of research have been performed [4-12]. It can be ex-

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pected that CQC will be one of the main approaches for near term applications of quantum computation [13-29]. However, only limited resources of CQC are available.

As developers, we report a newly implemented superconducting CQC platform, dubbed as ScQ (ScQ cloud quantum computation platform is available at <http://q.iphy.ac.cn>, which is updated at Quafu cloud quantum computation platform <http://quafu.baqis.ac.cn>), currently equipped with a 1D array of 10 superconducting qubits. Single-qubit rotation gates can be performed on each qubit. Controlled-NOT (CNOT) gates on pairs of nearest-neighbor (NN) qubits on the chain are available. To demonstrate the performance of ScQ, we use a well-accepted benchmark, multi-qubit Greenberger-Horne-Zeilinger (GHZ) states generation of up to all 10 qubits, to show the capabilities and programming of our platform. Generally, the prepared GHZ states and other entangled states can be used as a valuable resource for various quantum computation tasks and testing principles of quantum theory [30-43]. These results together with gate information and device parameters, will be useful for researchers on ScQ.

Performing tasks with CQC is different from that in laboratories. For example, online gate fidelities will be generally lower because we can optimize the operation accuracy in the laboratory for a fixed scheme, which may not be applicable for CQC. In this sense, online gate fidelities can only be op-

timized for general purposes. CQC is designed to efficiently produce highly uniform outputs. We also strive to make it easy to operate. By exploiting CQC, the generation and particularly verification of multi-qubit GHZ states of up to 10 qubits remain challenging, even if it is a widely used performance benchmark for various platforms of quantum computation.

2 Setup of ScQ

To set up the whole CQC platform, we allow users to access the web page of ScQ through a public network. Point-to-point communication is constructed between the lab computer and backend server for the web page. The framework of ScQ is summarized in Figure 1. First, the tasks of quantum circuits submitted by different online users are sent to a web server for syntax checking and task scheduling. Then, the tasks are translated to certified order strings and subsequently sent to the lab system. An agent service program is installed in the lab computer for constantly requesting tasks from the web server. Once a task is received, it will be performed by the agent via the lab system. After the computing of the quantum processing unit (QPU), the results are pre-treated by the lab computer and returned to the web server through the agent. Finally, the results are visually displayed

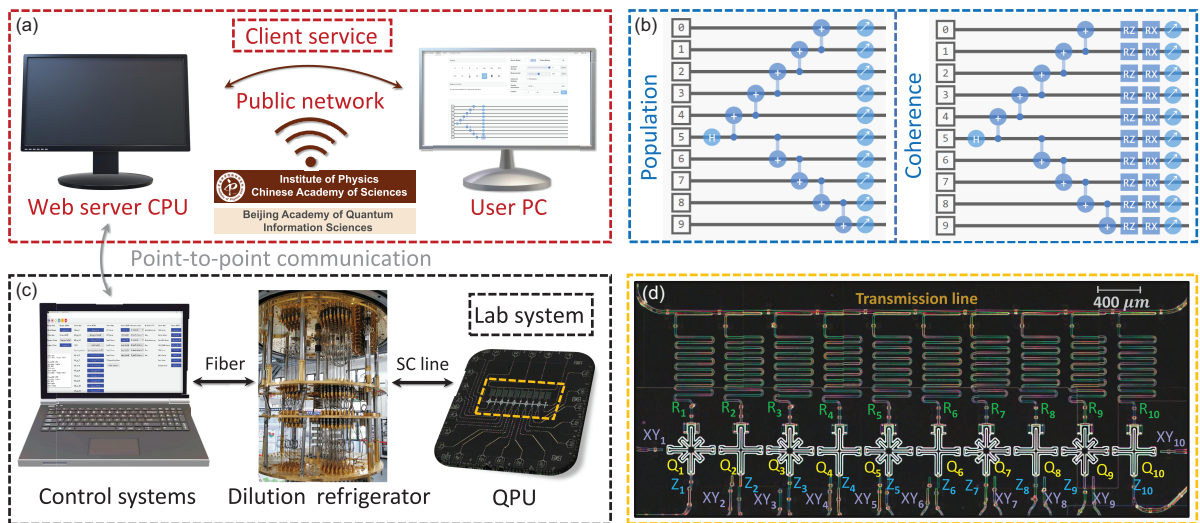


Figure 1 (Color online) Framework of ScQ. (a) Client service of ScQ. Users can visit the ScQ website to construct the quantum circuit and run it. The web server central processing unit (CPU) of ScQ interacts with users through the public network and connects to the experiment control systems in the laboratory via point-to-point communication. (b) Quantum circuits of measuring the population (left) and coherence (right) of GHZ states. (c) Automatic control of running quantum algorithms on the superconducting quantum processing unit (QPU) in the lab system. On the experimental computer, we deploy an agent server as an intermediary to receive commands from the web. The commands of quantum operations will be compiled into microwave pulses and uploaded to the electronics corresponding to control lines of qubits. (d) 10-qubit chain QPU of ScQ. Each qubit can be independently controlled by the XY control line (microwave) and Z line (flux bias). Ten readout resonators are coupled to the qubits, which are connected to the transmission line for quantum non-demolition (QND) measurement [44]. The QPU is mounted in a dilution refrigerator with the base temperature of the mixing chamber being approximately 10 mK.

on the web and users can download the results for individual data processing.

In the lab system, the QPU of ScQ consists of 10 qubits in a 1D array (see Figure 1(d) [44]). Considering the rotating wave approximation, the Hamiltonian of this system can be written as:

$$H/\hbar = \sum_i -\frac{1}{2}\omega_i\sigma_i^z + \sum_{i<j} g_{ij}(\sigma_i^+\sigma_j^- + h.c.), \quad (1)$$

where ω_i is the frequency of qubit Q_i , g_{ij} is the coupling coefficient of qubits, and $\sigma^\pm = (\sigma^x \pm i\sigma^y)/2$ is the raising (lowering) operator with $\sigma^{x,y,z}$ being Pauli matrices. The frequency of each qubit can be adjusted from 4.0 to 5.8 GHz. The NN qubits are directly coupled through capacitance, and the coupling strength is between 10 and 12 MHz. The coupling strength of the next NN qubits is one order of magnitude smaller than that of the NN qubits, which is approximately 1 MHz. All 10 qubits are coupled to one readout line through their individual readout resonators, the frequencies of which are between 6.49 and 6.66 GHz.

In the calibration procedure, we bias each qubit to its idle frequency using an independent Z control line. Taking into account the AC Stark effect between qubits, we diverge the idle frequencies of NN qubits by at least 418 MHz and the frequency gap is even larger during the readout process. Hence, the crosstalk can be suppressed when qubits are manipulated and measured. Based on this, we calibrate single- and two-qubit gates in the lab system. The pulse durations of single-qubit gates are uniformly set to 30 ns, and the two-qubit CZ gate is around 40-ns-long. The average randomized benchmarking (RB) [45-47] fidelity of single-qubit gates reaches 99.7%, whereas the average quantum process tomography (QPT) [48, 49] fidelity of two-qubit CZ gates reaches 95.5% (see [Supporting Information](#)).

All the above quantum gate operations can be remotely realized through the ScQ platform. Users can design their own quantum circuits using the drag-and-drop quantum gates toolbox of the graphic interface or with a quantum assembly (QASM) language of ScQ (<http://q.iphy.ac.cn>, <http://quafu.baqis.ac.cn>). Particularly, ScQ allows users to scan the arbitrary parameters of single-qubit rotation gates. By clicking the “Parameter Setting” button of a quantum gate on the graphic interface, one can determine whether to use the gate with a fixed rotation angle in general setting or with varying parameters in an advanced setting. ScQ also provides a QASM of coding in Python, which is similar to the Qiskit of IBM (<https://quantum-computing.ibm.com>). For instance, one can add quantum gates to manipulate qubits after initializing a `QuantumCircuit` object. In the following code, we show the basic imports and generation of a `QuantumCircuit` with 10 qubits:

```
import numpy as np
from numpy import pi
from qasm import QuantumCircuit
q = QuantumCircuit(10)
```

Here one can add gates to manipulate the qubits of `QuantumCircuit`, such as adding a R_x gate (`rx`) with a fixed angle $\pi/2$ on the qubit 0:

```
q.rx(0, pi/2)
q.measure([0])
result = q.send()
```

where the second line specifies the list of measured qubits, and the last line indicates that the task is sent to QPU and measurement result returns.

Compared with other CQC platforms, our ScQ provides a more direct way to express the “for” loop. For example, a R_x gate with varying angle from $-\pi/2$ to $\pi/2$ on the qubit 0 can be directly expressed as:

```
angles = np.linspace(-pi/2, pi/2, 51)
q.rx(0, angles)
```

where the second parameter of `rx` gate is a list with multiple angles. This “for” loop of rotation is completed by the built-in loop in the lab computer connected to the experimental QPU. In this way, the time cost of messaging tasks and data processing will be reduced, especially in tasks concerning the measurement of off-diagonal elements.

3 Online generation and verification of GHZ states

It is a performance standard for a quantum setup to produce multi-qubit entangled states with high fidelity. On the quantum processor of ScQ, we show the generation of high-fidelity GHZ states. Particularly, the GHZ states of all possible blocks of 1D array qubits from 6 to 10 qubits are prepared and verified, which provides a full figure of merit of the platform. The gate sequence for preparing a 10-qubit GHZ state is shown in Figure 1(b), which consists of a Hadamard gate and subsequent series of CNOT gates. Each CNOT gate is composed of a lab-native CZ gate and two single-qubit rotations. Note that different sequences for preparing GHZ states have different layers of CNOT gates. We use the circuit in Figure 1(b) to reduce the layers and crosstalk caused by parallel operations.

In the QASM-like expression, the generation and characterization of GHZ states corresponding to Figure 1(b) can be written as:

```

def ghz(n, angles = []):
    q = QuantumCircuit(n)
    ini = n // 2
    q.h(ini)
    for i in range(ini, 0, -1):
        q.cnot(i, i - 1)
    for i in range(ini, n - 1):
        q.cnot(i, i + 1)
    if angles:
        for i in range(n):
            a = [- j for j in angles]
            q.rz(i, a)
            for i in range(n):
                q.rx(i, pi/2)
    q.measure([i for i in range(n)])
    res = q.send()
    return res

num = 10
angles = np.linspace(-pi/2, pi/2, 51)
results = ghz(num, angles)

```

The above QASM code shows the characterization of the fidelity of 10-qubit GHZ states. The result of each loop is provisionally recorded in the lab computer and all results will return to the web once the whole experiment is completed.

Here, the fidelity of the generated GHZ state is defined as the distance between the prepared state and target state. The fidelity of a generated GHZ state should be above 0.5 [33], which ensures the genuine multi-qubit entanglement. We use the standard method to determine the fidelity, $F = (C + P)/2$, where P represents the summation of populations of states $|0\dots 0\rangle$ and $|1\dots 1\rangle$ corresponding to two diagonal elements of the density matrix and C denotes two off-diagonal elements corresponding to the relative coherence. We can obtain these two parameters experimentally. First,

we obtain the population P by making multiple measurements along the Z -axis and calculating the probabilities that the measurement results are in $|0\dots 0\rangle$ state and $|1\dots 1\rangle$ state, marked as $P(|0\dots 0\rangle)$, $P(|1\dots 1\rangle)$ respectively. Then we will get $P = P(|0\dots 0\rangle) + P(|1\dots 1\rangle)$.

In order to infer the relative coherence C , we need to introduce a rotation operator $\mathcal{P}(\gamma) = \bigotimes_{j=1}^N (\cos \gamma \sigma_{y,j} + \sin \gamma \sigma_{x,j})$ [38-41]. The corresponding parity, written as $\mathcal{P} = C \cos(N\gamma + \psi)$ will oscillate by varying γ . It is clear that the relative coherence C corresponds to the amplitude of \mathcal{P} . Moreover, the oscillation pattern of the parity \mathcal{P} depends on number of qubits N for the generated GHZ states. In the experiment, after the preparation of GHZ states, all the qubits should be rotated by the operation $\mathcal{P}(\gamma)$. Then, a Z -axis measurement will be applied, and the parity will be calculated $\mathcal{P} = \mathcal{P}_{\text{even}} - \mathcal{P}_{\text{odd}}$, where $\mathcal{P}_{\text{even}}$ and \mathcal{P}_{odd} correspond to the summation of all the probabilities of states with an even number of qubits and an odd number of qubits in the state $|1\rangle$, respectively. With the change in γ , which is realized by scanning the parameter feature of ScQ, we will obtain an oscillation curve with γ . The period of the oscillation curve is related to the number of qubits of GHZ states. By fitting the experimental \mathcal{P} with the cosine function, we will obtain the corresponding relative coherence C of GHZ states. We prepared GHZ states with qubit number from 6 to 10 with different combinations of qubits for all possible blocks in the 1D array qubits. The parity oscillation curve corresponding to each GHZ state is shown in Figure 2 and the results of fidelities are shown in Figure 3. The fidelity of 10-qubit GHZ state generated online reached 77.05%, which approaches the best records achieved in labs [39], ranging from 66.80% to 81.70% for the 10-qubit case [10, 37-39]. The gate sequences

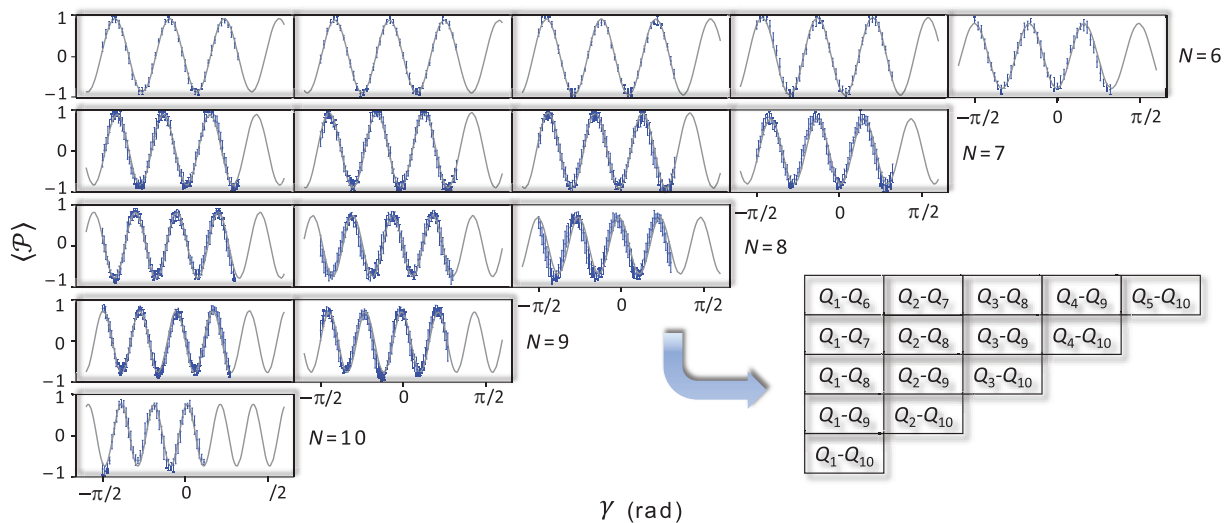


Figure 2 (Color online) Parity oscillations GHZ states with 6-10 qubits. Each row corresponds to GHZ states with the same number of qubits. From top to bottom, the qubits numbers of GHZ state are $N = 6, 7, 8, 9, 10$. From left to right, the GHZ state consists of the first N qubits and sequential N qubits, respectively.

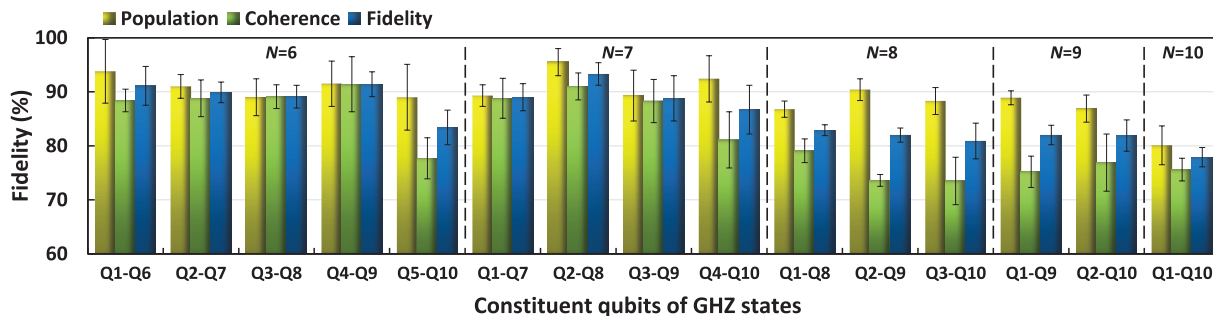


Figure 3 (Color online) Population, coherence, and fidelity of GHZ states with 6-10 qubits. From left to right, the qubit number of GHZ states are $N = 6, 7, 8, 9, 10$.

prepared for the GHZ states with different qubit numbers are shown in [Supporting Information](#).

4 Discussion and conclusion

In summary, we built an online CQC platform ScQ with 10 individually addressable superconducting qubits. The programmable QPU connected to the public network is available for every user to program arbitrary quantum circuits. Based on this device, we show how to use ScQ to generate and verify GHZ states. The preparation of GHZ states were performed on all possible blocks of qubits in the chain to provide a full evaluation of the performance of this platform. To facilitate a user-friendly experience, we allow users to manipulate qubits graphically or with a QASM programming language. The high controllability and efficiency of ScQ demonstrate the great potential of cloud computing architecture for creating new junctures in running quantum algorithms and studying quantum many-body problems. Although the results in current 10-qubit system can be obtained faster and of better quality on a classical computer with numerical simulations by software such as QuTiP [50], the experimental realization will be necessary when the number of qubits increases. It can be expected that a more advanced device with more qubits and higher control accuracy will facilitate the applications of quantum computation.

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Supporting Information

The supporting information is available online at <http://phys.scichina.com> and <https://link.springer.com>. The supporting materials are published as

submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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